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INFLUENCE OF ABSORPTION ON NONLINEAR VIBRATIONS OF GAS IN A CLOSED PIPE

R. G. Galiullin, E. R. Galiullina, and
E. I. Permyakov

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We consider dissipative mechanisms involved in resonance vibrations of gas in a closed pipe. Using analysis of a resonance curve as an example, we show the existence of four regimes differing in the mechanism of dissipation. We determine their boundaries, as well as lay a foundation for the procedures used to calculate the amplitude of vibrations within these intervals. Comparison of calculating formulas with experiments conducted by various authors is made.

Nonlinear vibrations in gas-filled tubes are excited mainly by a harmonically moving piston [1-6]. The regime in which the frequency of the vibrations of the piston approaches one of the resonance frequencies of the gas column is of greatest interest. The amplitude of pressure (speed) oscillations under these conditions is a complex function of the frequency and amplitude of the displacement of the piston, physical properties of the medium, and of the mechanisms of the dissipation of the energy supplied by the piston. At a distance from resonance, vibrations are described by the laws of linear acoustics [4]. Near resonance, there is a region of frequencies where vibrations become discontinuous: periodic shock waves are generated. Some authors assume that in this region the work of the piston goes for the generation of harmonics (nonlinear losses) and absorption in a laminar boundary layer near the wall [2-4]. At the same time, in some experiments a turbulent regime of vibrations was observed [1, 5]. Subsequently it turned out that transition to turbulence in a closed pipe was governed by a criterion that was an analog of the Reynolds number based on the acoustic boundary layer thickness [6]. We also note that in the regions of frequencies adjacent to the region of discontinuous vibrations the solution is described by continuous functions. This region may be termed the region with a comparatively weak nonlinearity. Then, on the axis of frequencies we can indicate several intervals within which the mechanism of dissipation remains invariable, while the transition from one interval to another is accompanied by a change in the mechanism of dissipation.

Below we attempt to analyze the resonance curve in order to determine the intervals and the amplitudes of vibrations within them.

To determine the region of discontinuities, the inequality

$$\delta < 4/\pi\,, \tag{1}$$

should be satisfied [7], where $\delta = 2\pi\Delta(l_p/2L)^{1/2}$, $l_p = c_0^2/\varepsilon\omega v_1$ is the distance of the formation of a discontinuity; Δ is determined from the relation $\pi\Delta = k_0L - kL$; here $k_0 = \omega_0/c_0$, $k = \omega/c_0$; the subscript "0" refers to the exact resonance. If we take into account that with this resonance $k_0L = \pi$, then $\Delta = (k_0L - kL)/\pi$ is the relative maladjustment. Let us introduce the absolute maladjustment $D(kL) = k_0L - kL$; then we can show that Eq. (1) is equivalent to the relation

$$\frac{|\Delta\,(kL)|}{kL} \le \frac{2}{\pi}\left[(\kappa+1)\frac{l}{L}\right]^{1/2}, \tag{2}$$

from which we can easily obtain the values of $(kL)_C$ and $(kL)_{C'}$ given in Fig. 1, where the resonance curve is presented. They correspond to the boundaries of the interval kL within which discontinuities are formed. Under

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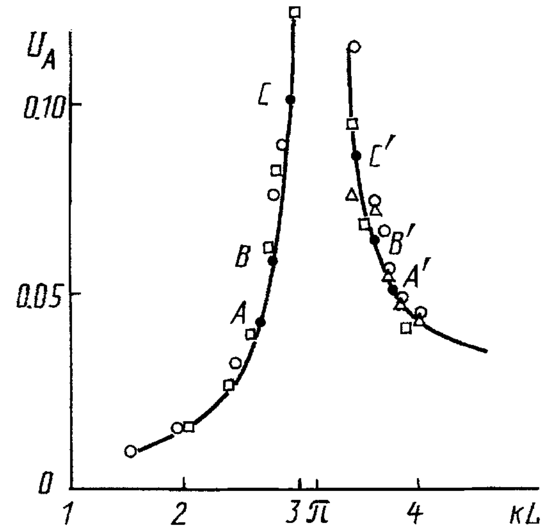


Fig. 1. Relationship $kL - U_A$ (resonance curve). Points, experiments [5]; curves, calculations by formula (2).

the conditions of work [5], where $\kappa = 1.4$, $l/L = 8.12 \cdot 10^{-3}$, we have $(kL)_C = 2.88$, $(kL)_{C'} = 3.47$. The agreement with the experiment is within 1%.

To determine the boundaries of the acoustic region, it is necessary to have a solution that takes into account the second approximation. Let us avail ourselves of the expression from [7]:

$$U(a, t) = M \frac{\sin k(L-a)}{\sin kL} \sin \omega t - \frac{\varepsilon M^2 k(L-a)}{4 \sin^2 kL} \cos 2k(L-a) \cos 2\omega t, \qquad (3)$$

where $M = \omega l / c_0$; a is the Lagrangian coordinate of a particle; U is the dimensionless velocity. Let us consider expression (3) at the point $a = L - \pi/(2k)$; here the first harmonic has the maximum amplitude

$$U_{1m} = \mathrm{M}/\sin kL, \qquad (4)$$

while the amplitude of the second harmonic takes the value $U_2 = \varepsilon/8 \cdot (M^2 \pi / \sin^2 kL)$. If we impose the requirement that on the boundary of the acoustic region the amplitude of the second harmonic should not exceed 2% of the first one (a conventional requirement to the frequency of a signal in radioelectronics), then the boundaries of the acoustic region can be found from the condition

$$\frac{kL}{|\sin kL|} = \frac{4}{2.5 \pi \varepsilon (l/L)}. \qquad (5)$$

If we assume that, as before, $l/L = 8.12 \cdot 10^{-3}$, then $(kL)_A = 2.61$, $(kL)_{A'} = 4.02$. In Fig. 1 the acoustic regions will be located to the left of the point A and to the right of the point A'. In these regions we may discard the second term on the right-hand side and calculate the maximum amplitude from formula (4).

It should be noted, however, that expression (4) does not take the wall absorption into account. It can be accounted for if we introduce the complex wave number $k^* = k[(1 + \beta') + i\beta'']$, where β' determines dispersion and β'' absorption. In the case of high-frequency vibrations $H = R\sqrt{\omega/2\nu} \gg 1$, where $\beta' = -\beta'' = 0.5(1 + (k-1) / \sqrt{Pr})H^{-1} \ll 1$ (Pr is the Prandtl number [8]). Then, instead of Eq. (4) we have

$$U_{1m} = \frac{\mathrm{M}}{\sqrt{\cos^2 kL \,\mathrm{sh}^2 \beta + \sin^2 RL \,\mathrm{ch}^2 \beta}}; \qquad (6)$$

where $\beta = kL\beta'$ [9].

The transition to turbulence is determined by the criterion $A_C = 2U_A/(\omega\nu)^{1/2}$, where U_A is the amplitude of the vibrations of speed. In closed pipes turbulization sets in when $A_C \geq 400$, if the boundary layer is thin in comparison with the pipe radius, i.e., when $H \gg 1$ [6]. The occurrence of turbulence during vibrations is a local phenomenon: in a pipe there may coexist portions of laminar and turbulent motions. In view of the fact that in a closed pipe the antinode of speed is located near the middle of the pipe, turbulence also appears primarily there [6].

By virtue of the fact that under the conditions of work [2] the maximum possible magnitude of A_C does not exceed 380, and under the conditions of work [1] it is not smaller than 2300, then in the former case the flow seems to be laminar at all of the frequencies of piston vibration, while in the second case it is turbulent. Thus, experimental facilities should be subdivided into those in which laminar flow is preserved and those in which turbulence is generated.

In the facilities of the first type the amplitude in the region of discontinuities can be calculated by the procedure suggested in [2], whose idea is that the energy supplied by a piston into a pipe is equal to the losses on the pipe wall and for the generation of harmonics:

$$\langle \dot{E}_{\mathrm{p}} \rangle = \langle \dot{E}_{\mathrm{w}} \rangle + \langle \dot{E}_{\mathrm{n}} \rangle , \tag{7}$$

where $<\dot{E}_{\mathrm{p}}>$ is the amount of energy averaged over time supplied by the piston to the pipe per unit time; $<\dot{E}_{\mathrm{w}}>$, $<\dot{E}_{\mathrm{n}}>$ are the rates of energy dissipation due of the wall losses and nonlinear losses, respectively. Assuming that the amplitude of the oscillations of pressure is identical over the entire pipe length, we can obtain [2]:

$$\langle \dot{E}_{\mathrm{p}} \rangle = \rho_0 c_0^3 \frac{l}{L} U_m , \quad \langle \dot{E}_{\mathrm{w}} \rangle = \frac{1}{2} \beta \rho_0 c_0^3 U_m^2 , \quad \langle \dot{E}_{\mathrm{n}} \rangle = \frac{1}{12} (\kappa + 1) \rho_0 c_0^3 U_m^3 , \tag{7'}$$

where $U_m = 2U_A$ is the magnitude of the discontinuity of the speed related to the speed of sound. Substituting Eq. (7′) into Eq. (7), we obtain an expression for U_A that coincides with the experiment qualitatively, but differs from it by a factor of two. Allowance for the real form of vibrations leads to the appearance of the factor 2/3 in the expression for $<\dot{E}_{\mathrm{p}}>$ [1]. This is, however, insufficient for explaining the substantial difference between the theory and experiment.

Let us consider the correctness of the expression for $<\dot{E}_{\mathrm{w}}>$. The wall losses are in effect thermoacoustic heat fluxes [10]. They lead to nonuniform heating of the pipe wall. In the case of harmonic vibrations the heat flux on the wall has the form

$$\langle q_1 \rangle = \frac{1}{8} \rho_0 c_0^2 U_1^2 \sqrt{2\nu\omega}\, (A \cos 2 (kx + \alpha_0) + B \operatorname{ch} 2\beta^*) ,$$

where U_1 is the dimensionless amplitude of vibrations; A, B, α_0, β^* are constants. In the case of exact resonance in a closed pipe $\alpha_0 \to 0, \beta^* = \beta << 1, B = (\kappa - 1)/\sqrt{Pr} + 1$. We can easily see that $Q_1^* = 2\pi R \int_0^L <q_1> dx$ are the wall losses and $Q_1 = Q_1^*/\pi R^2$ are the losses per unit cross-sectional area of the pipe. Having performed integration, we obtain

$$\langle \dot{E}_{\mathrm{w.1}} \rangle = \frac{\beta}{2} \rho_0 c_0^3 U_1^2 .$$

In the case of discontinuous vibrations of the form [11]

$$V = U_{\mathrm{A}} \sum_{n=1}^{\infty} \frac{2}{n (1 + \sigma)} \sin n\omega t ,$$

where $\sigma = 1.57$ is the distance of the stabilization of a sawtooth wave, assuming that the contributions of thermal effects of separate harmonics $\omega_n = n\omega$ are additive [2], we can obtain

$$\langle \dot{E}_{\mathrm{w}} \rangle = \frac{\beta}{2} \rho_0 c_0^3 \sum_{n=1}^{\infty} \frac{1}{n^{3/2} (1 + \sigma)} U_A^2 . \tag{8}$$

By virtue of the fact that the sum of the series is equal to unity with accuracy to 1%, we write

$$\langle \dot{E}_{\mathrm{w}} \rangle = \frac{\beta}{2} \rho_0 c_0^3 U_A^2 . \tag{8'}$$

Comparison of Eq. (8′) with $<\dot{E}_{\mathrm{w}}>$ in Eq. (7′) shows that they differ by a factor of four.

Let us consider nonlinear losses. The rate of energy dissipation in a weak shock wave is equal to $<\dot{E}_n> = - T_0 <\dot{S}>$ [12], where T_0 is the temperature of the medium prior to the jump; $<\dot{S}>$ is the rate of change in the total entropy of the system; < > denotes averaging over the period of vibrations. Let us express $<\dot{S}>$ in terms of the jump in the specific entropy for the period of vibrations ΔS:

$$\langle \dot{S} \rangle = \frac{\rho_0}{\tau} \int_0^V \Delta S \, dV , \tag{9}$$

where ρ_0 is the density; τ is the period of vibrations. Owing to [12]

$$\Delta S = - \frac{1}{12 T_0} \frac{\kappa + 1}{\rho_0 \kappa^2 p_0^2} (p_2 - p_1)^3 ,$$

where $(p_2 - p_1)$ is the pressure jump, and p_0 is the mean pressure in the pipe, for the unit cross-sectional area we obtain

$$\langle \dot{E}_n \rangle = \frac{1}{12\tau} \frac{\kappa + 1}{\kappa^2 p_0^2} \int_0^L (p_2 - p_1)^3 \, dx . \tag{10}$$

In the case of a constant amplitude of vibrations along the length of the pipe, with account for $\tau = 2L/c_0$, $\kappa p_0 = \rho_0 c_0^2$, $p_2 - p_1 = U_m \rho_0 c_0^2$, it is possible to find a magnitude of $<\dot{E}_n>$ half as large as $<\dot{E}_n>$ in Eq. (7').

The assumption about the constant amplitude of vibrations is not fulfilled in experiments. Thus, in the tests of [5] the amplitude ratio of pressure near oscillations the piston and in the middle of the pipe amounted to 4, while in [13] it was equal to 4.3. Since no analytical relationships have been obtained as yet for the vibrations of amplitude along the length of the pipe, we processed the experimental data of [5]. This processing made it possible to express the rate of nonlinear dissipation in the form

$$\langle \dot{E}_n \rangle = \frac{\kappa + 1}{3} m \rho_0 c_0^3 U_m^3 , \tag{11}$$

where for the conditions of [5] $m = 0.355$. With account for the corrections made, we can write a formula for computing the amplitude of vibrations:

$$U_A = \sqrt{\left(\gamma^2 + \frac{4}{m(\kappa + 1)} \frac{l}{L} \right)} - \gamma , \tag{12}$$

where $\gamma = 3\beta/4m(\kappa + 1)$.

The dependence of the dimensionless amplitude of pressure oscillation on the relative amplitude of the displacement of the piston in a laminar regime of vibrations is presented in Fig. 2. Here, the points represent the experimental results of various authors; the solid line is the result of calculation by expression (12) with account for $\delta p/p_0 = \kappa U_A$. The parameter β corresponds to the conditions of the setup in [2]. It is seen that all of the points, except for one, are located on the calculated line.

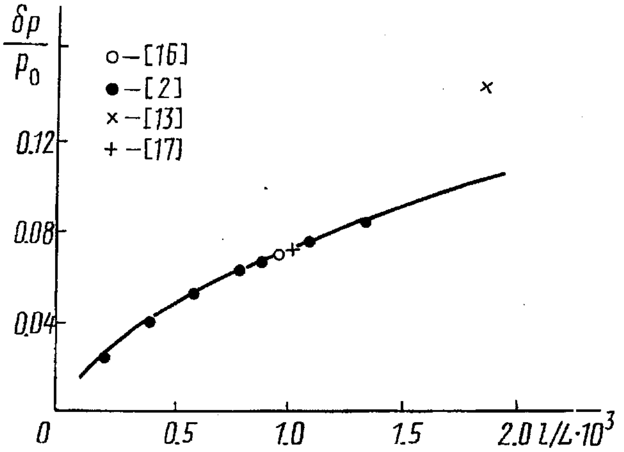
In the region of weak nonlinearity (the arcs AC and $C'A'$ in Fig. 1), the solution is continuous and is described by the expression of form (3). At the point $a = L - \pi/2k$ the amplitude ratio of the harmonics $\sigma = U_2/U_1$ is determined by the formula

$$\sigma = \frac{\varepsilon}{8} \frac{\pi k L}{\sin kL} \frac{l}{L} .$$

By virtue of the fact that on the boundary of the region of discontinuities $\sin kL \sim \Delta(kL)$, where $\Delta(kL)$ is determined by relation (2), the value of σ may become larger than

$$\sigma_m = \frac{\pi^2}{32} \left[(\kappa + 1) \frac{l}{L} \right]^{1/2} . \tag{13}$$

The value of σ_m does not exceed 5% at $l/L = 8.12 \cdot 10^{-3}$. Further we can show that the contribution of the second harmonics to the resultant amplitude will be still smaller, therefore for the calculation of the amplitude we



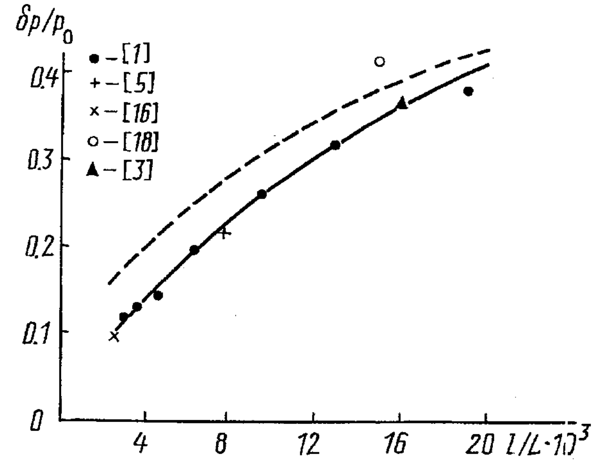


Fig. 2. Dependence of the dimensionless amplitude of pressure oscillations $\delta p/p_0$ on the relative amplitude of the displacement of a piston l/L in a laminar mode of vibrations.

Fig. 3. Dependence of the dimensionless amplitude of pressure oscillations $\delta p/p_0$ on the relative amplitude of the displacement of the piston l/L in a turbulent regime of vibrations.

can use Eq. (4) or, with account for the wall absorption, Eq. (6). In Fig. 1 the points represent experiments [5]; the solid curves represent calculation by formula (6). The agreement is also good.

Let us determine the band of frequencies within which the flow is turbulent. If turbulence appears beyond the boundaries of the region of discontinuous vibrations, then for the solution of the problem it is sufficient to express the amplitude of vibrations in terms of A_C and substitute the expression obtained into Eq. (4). Then

$$\frac{\sqrt{kL}}{\sin kL} = \frac{A_c}{2}\sqrt{\left(\frac{\nu c_0}{l^2 L}\right)}. \tag{14}$$

Solution of (14) for kL yields roots the first two of which determine the boundary of the transition. Under the conditions of work [5], i.e., $A_C = 400$, $l = 13.8\cdot 10^{-3}$ m, $L = 1.7$ m, we have $(kL)_B = 2.71$, $(kL)_{B'} = 3.64$ (Fig. 1). When $(kL)_B > (kL)_C$, $(kL)_{B'} < (kL)_{C'}$, determination of the points of transition seems to be difficult. In this case we can solve the problem of the regime of vibrations if we compare the amplitude of the vibrations expressed in terms of A_C with its value from relation (12) or from the experiment. If the first one is higher than the second, then in the region of discontinuities vibrations are realized in a laminar regime and in the opposite case in a turbulent regime.

However, it should be emphasized that the criterion of transition depends substantially on the state of the wall surface, foreign inclusions, etc. Thus, the introduction of the probe of a thermoanemometer into the boundary layer led in [6] to the reduction of A_C from 400 to 300. Apparently, it is possible to apply special measures to attain higher values of A_C. Thus, in [14] the magnitude $A_C = 800$ was attained. When analyzing the regime of vibrations, it is advisable to take this circumstance into account, i.e., only those vibrations are to be considered turbulent in which the experimental amplitude of the vibrations is higher than the amplitude calculated from the condition $A_C = 800$.

Table 1 lists the parameters of the setup (the 1st and 2nd columns), dimensionless amplitude of pressure oscillations (3rd column), dimensionless amplitude of pressure oscillations during transition to turbulence (4th column), and analysis of the regime (5th column). It is seen that the inference about the regime of vibrations can be made with certainty.

To obtain an expression similar to Eq. (12), it is necessary to determine turbulent losses at the wall. Investigation of turbulent vibrating flows was the concern of a number of works that can be found in review [14]. It is found that turbulence with vibration has a high-frequency character; moreover, the lowest frequency of pulsations can exceed the frequency of vibrations by an order of magnitude [1]. The compressibility of the medium

TABLE 1. Experimental Data

Reference	$l \cdot 10^3$, m	L, m	p_m/p_0	$(p/p_0)_{tr} \cdot 10^2$, $A_C = 800$	Regime of vibrations	$H = R\sqrt{\omega/2\nu}$
[1]	39	2.4-14	0.35-0.10	(14.74-5.58)	Turbulent	169-64
[2]	0.2-2.0	1.5	0.018-0.08	15.2	Laminar	–
[3]	55	3.4	0.36	11.3	Turbulent	133
[5]	13.8	1.7	0.20	14.4	"	43
[15]	3.0	3	0.07	10.8	Laminar	–
[16]	3.4	12	0.10	5.4	Turbulent	60
[13]	3.2	1.7	0.14	14.4	Laminar	–
[17]	1.8	1.7	0.07	14.4	"	–
[18]	49	3.2	0.41	11.7	Turbulent	127

does not exert a substantial effect on turbulence [14]. If we integrate the boundary layer equation for an incompressible fluid over the pipe cross section, then the pressure losses Δp can be expressed as [14]:

$$\frac{\Delta p}{l} = \rho_0 \frac{dU}{dt} + \frac{4\tau_w}{d}, \tag{15}$$

When all the quantities entering into Eq. (15) vibrate in identical phases, the flow can be considered to be quasistationary. Analysis of Eq. (15) with account for phase shifts shows that, when $0 < H \leq 0.94$, the flow can be considered quasistationary and, when $H \geq 20$, the inertia term in Eq. (15) substantially exceeds turbulent losses at the wall [14]. It should be noted that the condition $H \leq 0.94$ is more rigorous than the expression $z = 4R\omega/\lambda_s \overline{U}$ [19]. The fifth column of Table 1 presents the value of H under the conditions of experimental works discussed in the present article. It is seen that all the data were obtained for $H > 20$.

In the case of vibrating flows, the rate of regular vibrations is superimposed by turbulent pulsations, i.e., $u = U(t) + u'$, $v = V(t) + v'$, where u and v are the axial and normal components of the speed, respectively; U and V are the regular portion of the vibrations of speed, therefore the turbulent friction τ_t contains three components $-\tau_t = \rho_0 \overline{Uv'} + \rho_0 \overline{Vu'} + \rho_0 \overline{u'v'}$.
In the case of a high-frequency turbulence, when regular components have no time to change substantially, the first two terms vanish, then $\tau_t = -\rho_0 \overline{u'v'}$, τ_t is determined in the same way as in a steady flow. This conclusion was verified experimentally [14].

To describe turbulent losses, we shall introduce the turbulent coefficient of absorption β_t. The procedure of its determination consists in the linearization of the expression $\tau_W = (\lambda_s/8)\rho_0 \overline{U}^2$.Then [20]

$$\beta_t = \frac{\lambda_\xi L}{6R} \overline{U}. \tag{16}$$

Let $<\dot{E}_w>_t$ be the rate of energy dissipation due to turbulence. By defination [12] $<\dot{E}_w> = \beta 2 c_0 \overline{E}$, $<\dot{E}_w>_t = \beta_t 2 c_0 \overline{E}$, where $\overline{E}$ is the mean density of energy in the pipe, we have $<\dot{E}_w>_t = (\beta_t/\beta)<\dot{E}_w>_t$ and, with account for Eq. (8'), $<\dot{E}_w>_t = 1/2\beta_t \rho_0 c_0^3 U_A^2$. Having substituted this expression into the energy balance equation (6), we easily obtain the expression

$$U_A = 4\sqrt{\left(\frac{l/L}{\lambda_s (L/R) + 4m(\kappa + 1)}\right)} \tag{17}$$

for calculating the amplitude of vibrations in a turbulent regime. For rough pipes $\lambda_s \approx const$, and Eq. (17) is final. In the case of smooth pipes $\lambda_s = 0.316\mathrm{Re}_s^{-0.25}$, $\mathrm{Re}_s = \overline{U}d/\nu$. We find the amplitude of vibrations from the equation

$$4m(\kappa + 1) U_A^2 + C U_A^{1.75} - 16(l/L) = 0, \tag{18}$$

where $C = 0.316(c_0 d/\nu)^{-0.25}(L/R)$, $m = 0.355$.

The dependence of the dimensionless amplitude of pressure oscillations $\delta p/p_0$ on the relative amplitude of piston displacement l/L is presented in Fig. 3, where the points represent the experimental results of various authors, the solid curve represents theoretical calculation by formula (18) with the parameters of the setup of [1], the dashed line shows the calculation in conformity with Eq. (17) at $\lambda_s \equiv 0$ (with account for nonlinear losses alone). It is seen that all the experimental points (except for one) group around the solid curve. We can note that the most substantial contribution (up to 60%) is made by the wall region at relatively small values of l/L. As l/L increases, its influence decreases to 9-10% .

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NOTATION

L, length of the pipe; R, radius of the pipe; d, diameter of the pipe; ν, coefficient of kinematic viscosity; c_0, speed of sound in an unperturbed gas; $\varepsilon = (\kappa + 1)/2$, parameter of nonlinearity; $\kappa = c_p/c_v$, where c_p and c_v are the specific heats at a constant pressure and constant volume, respectively; ω, cyclic frequency; $k = \omega/c_0$, wave number; l, amplitude of the vibrations of piston; $M = \omega l/c_0$, Mach number; Re, Reynolds number; v, amplitude of the speed of the piston; U, dimensionless rate of vibrations; U_{1m}, maximum value of the amplitude of the 1st harmonic; U_A, amplitude of speed oscillations; $\overline{U}$, pipe cross-section-averaged speed; $H = R\sqrt{\omega/2\nu}$, frequency parameter; $A_C = 2U_A/(\omega\nu)^{1/2}$, the Sergeev number; τ_w, shear stress on the wall; λ_s, coefficient of hydraulic resistance; β_t, turbulent coefficient of absorption. Subscripts 1 and 2 relate to the 1st and 2nd harmonics, respectively.

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